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# The non-linear income-growth relation

## A new look at a much analyzed relation

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Abstract:

The paper considers all pairs of initial income and growth in the updated Maddison data from 1950 to 2010. The analysis confirms the well-known fact that the income-growth relation has low explanatory power, but when it is analyzed by kernel regression techniques, a highly significant hump-shaped relation does appear. It gives the long-run dynamics of the incomes of the countries of the world. Further, a moving measure of variation is developed for the growth rate. It is shown that the income-variation relation has a large fall from middle income to high income.

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## 1. The income-growth relation

Few relations in economics have been researched as much as the income-growth relation. It considers initial income and growth ($y_{i(-)}$, $g_i$).[3] The relation is often illustrated by a diagram with initial income at the horizontal axis and growth at the vertical one. In large data-sets the ($y_{i(-)}$, $g_i$)-pairs scatter wildly, but the scatter hides an 'underlying' income-growth relation that points to the transition from a traditional to a modern steady state.

The paper studies the relation by kernel regression. This technique has two advantages: (a1) It does not require a model from economic theory. Therefore, it reaches results that all growth theories should be able to replicate. (a2) It works with stacked and sorted data, so that all other country differences than income are randomized.

These advantages come at two disadvantages. (d1) The study merges data across time and countries. This assumes *equivalence*: Long-run and cross-country data tell the same story. (d2) The kernel-technique is a semi-graphical univariate technique. Obviously the world has more dimensions. However, we consider a near-consistent set of $N$ = 8,874 ($y_{i(-)}$, $g_i$)-pairs of observations. This is enough data so that it could be broken up in many ways, giving some insight in other dimensions.

The kernel has a robust path, which proves highly non-linear. Under the equivalence assumption the path gives the long-run dynamics of the population of countries in the world. Hence, it does answer some much discussed questions, of which we look at two: The first is if a *low level equilibrium trap* exists. This is the case if the relation has a low income interval, where the curve has a negative slope and falls below the y-axis. The second question deals with *convergence*: Do the poor countries catch up with the rich ones? It is the case if the income-growth relation has a negative slope. Convergence may occur throughout or locally at some income interval.

The content of the rest of the paper is: Section 2 looks at the theories. Section 3 describes the data and argues that they are rather representative. Section 4 looks at the pattern in the data. Section 5 considers the development in the variation in the data. Finally, section 6 concludes. This paper is written on the basis of a longer paper, which documents many additional results, see Paldam (2015).

3. The data comes from a ($y_{jt-1}$, $g_{jt}$)-panel, where $j$ and $t$ are indices for country and time. The data is stacked and sorted to the ($y_{i(-)}$, $g_i$)-data set where $i$ is an index for the income order. Time only appears as (-) indicating that each data pair is from the same country and the two time periods $t$ and $t$-1. In the literature $g_i$ is often an average $g_{ni}$, where $g_{njt} = (g_{jt} + g_{jt+1} + ... + g_{jt+n-1})/n$. See the appendix on definitions.

## 2. Some theory

The theory and empirics of economic growth is a large field, covered by numerous textbooks, and a massive four volume handbook (Aghion and Durlauf 2005, 2013). Thus, a few notes on a couple of central themes of relevance will suffice. For ease of presentation the World Bank terminology of LICs, MICs, and HICs is used. It refers to Low, Middle, and High Income Countries respectively. Income is defined as the logarithm to GDP per capita, in fixed PPP prices; see the appendix.

### *2.1 Growth regressions*

Much of the discussion of the ($y_{i(-)}$, $g_i$)-relation starts out from the empirical observation that long run time series for income, *y*, often look amazingly linear – perhaps with one or two kinks. This is often taken to represent a Solow model growing along a steady state path.

From this theoretical backbone a large wave of growth regression was started by Barro (1991) and the ensuing textbook Barro and Sala-i-Martin (1995, 2003). It demonstrates how the model leads to a basic estimation model in two versions:

(1) $g_{njt} = \alpha_1 + \beta_1 y_{jt\text{-}1} + \varepsilon_{1jt}$, the ε's are the noise terms

(2) $g_{njt} = \alpha_2 + \beta_2 y_{jt\text{-}1} + [\gamma_1 z_{1jt} + \ldots + \gamma_k z_{kjt}] + \varepsilon_{2jt}$, the *z*'s are *k* controls

Indices *j* and *t* are for countries and time, while *n* indicates an average over *n* years.

Equation (1) tests for *absolute* convergence. It occurs if $\beta_1 < 0$. The standard result is that $\beta_1 > 0$ but insignificant, so convergence is rejected.[4] Most data sets used to estimate (1) give quite wild point scatters, so the F-test for the regression (1) is often insignificant, and $R^2$-scores are low indeed.

Equation (2) tests for *conditional* convergence, which occurs if $\beta_2$ turns significantly negative for a reasonable and robust *z*-set of controls for country heterogeneity. It already happens when the *z*-set is fixed effects for countries. Thus, countries would converge, if they were the same. It is almost a tautology, and it is surely not a statement about the real world. However, it is fairly easy to find more interesting control-sets making $\beta_2$ negative, as already reported in Barro (1991).

These equations have been estimated in many ways and on many data sets, both in

4. In our set of $N$ = 8,874 observations $\beta_1$ becomes significantly positive. The divergence is 0.15 per logarithmic point, or 0.75 percentage points over the 5 point income range (see Figures 2-4). It will be shown that this is a misleading picture.

pure cross-country versions (where $t$ is fixed) and in panel versions (where both $t$ and $j$ varies).[5] About 400 $z$-variables have been tried in many combinations,[6] and the relation has been adjusted for simultaneity using many sets of instruments. Many $n$'s have been examined, lags have been included, etc.

Since Baumol (1986) it has been known that the HICs converge to the *same* steady state. This is termed club convergence. It is dubious if other clubs than the HIC club exists.

### *2.2 Two steady states and the Grand Transition*

Ever since Rostow (1960) economic historians have pointed out that the most common steady state throughout history has been a traditional one with low income and no growth. This is confirmed by the large efforts of data collections put together by Maddison (2001). It is a steady state in the sense that it is stable, which is due to very slow technical progress. It is a LIC club if a low level equilibrium trap exists, so that the LICs converge to the *same* (low) level.

Thus, it is a basic observation that two steady states exists: A modern and a traditional. The modern started to develop about 200 years ago. The present HICs gradually moved away from their old traditional level and in the last half century converged to much the same income level. There is still a group of LICs, but it is debated if there is convergence at the bottom. If the top of the distribution grows faster than the bottom, the 'bridge' between the two keeps growing. We term the bridge the *Grand Transition* as it consists of many transitions.

Nearly all socio-economic variables have different levels in the traditional and modern steady state and a fairly well-defined path as the country moves between the two states.

(3) $z = F(y)$, with the levels $z^T$ in the traditional and $z^M$ in the modern steady state.

In other work we have shown how the transition paths look for some transitions.[7] Equation (3) models the causal effect $y \Rightarrow z$, while equation (2) models the causal effect $z \Rightarrow g \approx \Delta y$. So that estimates of both (2) and (3) have to be adjusted for simultaneity.

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5. See Gundlach and Paldam (2015) on the interaction of growth theory and the many estimators tried.
6. It is only possible to include a handful or two of the 400 variables so billions of choices are possible. Each of these gives an estimate of $\beta_2$, resulting in a large range of possible estimates. See chapter 12.5 of Barro and Sala-i-Martin (2005).
7. The reader will think of the agricultural and the demographic transitions. We have shown that there is also a democratic transition (see Gundlach and Paldam 2009b and Paldam and Gundlach 2012), a religious transition (see Paldam and Gundlach 2013), a transition of corruption (Gundlach and Paldam 2009a), and a transition in the preferences for socialism (Bjørnskov and Paldam 2010).

The definition of a steady state is that all ratios in the economy stay constant, and this is precisely what they fail to do! When the bridge between the LIC and the HIC steady states is a transition, with structural changes in all fields, it becomes rather unclear if this leads to a linear aggregate path. Large-scale structural change creates all kinds of tensions and uncertainties. Political alliances break up and new have to be formed. This may involve changes of the political system. It follows that it is likely that: (i) Growth becomes more variable during the transition. Also, the transition is the period where some MICs catch up with the HICs, i.e., technological catch-up may happen. Once that starts, it is likely that (ii) growth can become particularly fast. So we look for a middle income peak. From this peak to the high end of the income scale, the income-growth curve must have a negative slope resulting in the HIC-club.

### *2.3 Growth models with two sectors*

One way to catch some of the transition is by a two sectors model with a traditional and a modern sector. In the beginning all is traditional, and then the transition is shown as a process where the modern sector gradually replaces the traditional one. This idea goes back to Lewis (1954) and Ranis and Fei (1960), and many formalizations have been made. The model has also been generalized to more sectors. A key feature of these models is that early development leads to large gaps in productivity between the sectors, giving rise to strong tensions in the economies. As development progresses, the gaps close and tensions are reduced.

This model catches a growth process that starts slowly as the modern sector has no weight, but then it becomes gradually more important and able to account for more growth. In the process large differences between the productivity in the sectors emerge. That may lead to large social tensions causing growth to stall in the middle, so that the variability is likely to be much higher in MICs than in the HIC, where a steady state has been reached.

However, even when the two-sector model may catch some of the transition, the Grand Transition is a much more comprehensive process: It includes a large fall in corruption, democratization, a large fall in religiosity, etc. These changes contribute to growth, but they may not be smooth, so they may add political instability that affects investment.

### *2.4 Two cases: Old west and oil countries*

The transition started 200 years ago in a group of ‘old’ western countries, who are the largest group of HICs today. The transition was a gradual process, so it is easier to describe as a log-linear process, where technical progress was randomly distributed throughout the economy. The growth process in these countries will be termed the case of the *old west*. They never had as large tensions between the sectors as the MICs today. This is a qualification to equivalence

All countries earn some resource rent. It is typically 1 to 2% of GDP only, but some countries have abundant resources and resource rent becomes a major part of GDP. Such countries may reach HIC income levels exclusively from resource rent, without going through the Grand Transition. For long they remain LICs in the structure of society and its institutions, but gradually they start to change structure, to conform to income. The most extreme version of this case of development is the oil countries as an oil sector is a small international enclave in the country, employing few people and using a foreign technology. Such sectors are normally carefully fenced and heavily guarded so they are indeed an enclave. Their effect on the rest of the economy is the tax on the resource rent that flows into the treasury. We use OPEC membership as our proxy for the oil-case.

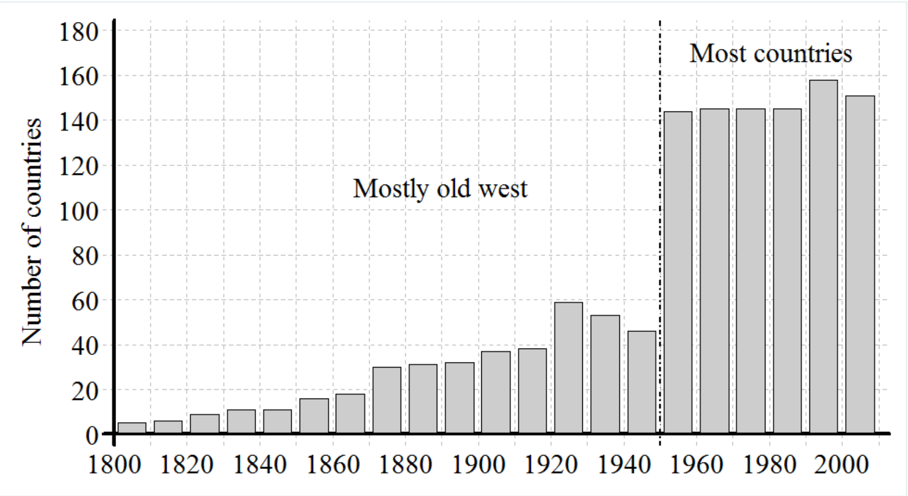
## 3. The data: ($y_{i(-)}, g_i$) pairs

The data used is from the Maddison-project where the annual data starts in 1800 and (p.t.) ends in 2010. We include data for the present countries as soon as possible. Data for Sub-Saharan African starts in 1950, where all but three counties were colonies. Data for the successor states of Yugoslavia starts in 1952, while data for the successor states of the USSR starts in 1990, and so they does for the successor states of Czechoslovakia. In these cases the data for the 'old' country is stopped, when the new data starts.

The data covers 12,786 data-pairs. Figure 1 gives the average number of countries covered in each decade. During the 19th Century 74% of the observations are from the old west, then some Latin American countries join the countries covered, but before 1950 the data is from a shifting sample of little representativity.[8] From 1950 the sample holds at least 144 countries with more than 95% of the world population. Thus, the 1950-2010 data sample is much more representative. The present study only covers this sample, where $N$ = 8,874.

Income is in (natural) logarithms to 1990 international Geary-Khamis dollars. They cover the interval from 5.7 to 10.4. This is 4.7 ***lps*** (logarithmic points) which is a span of 110 times in the *gdp*. To close such a gap in a century means that an excess growth of 4.8% above the one of the rich countries. This is next to impossible.

Figure 1. The coverage of the data




8. Till 1900 only three LICs are included (Indonesia 1815, Sri Lanka 1870, and India 1884). A few more are included before 1950, but the country sample from 1800 to 1950, where N = 3,912, is very skew. Also, national accounting for most countries only started in 1950, so data before that are backward projections. The data from 1800 to 1950 are analyzed in Paldam (2015).

## 3. The scatter of the ($y_{i(-)}$, $g_i$)-points analyzed by kernel-curves

The data-pairs are shown as a scatter diagram in Figure 2. The points scatter very much, so it is difficult to see any pattern. The wild scatter means that any simple pattern, such as the one discussed, will explain a modest part of the variation only. Figure 2 is provided with a kernel regression using the Epanechnikov kernel (E-kernel), with a bw (bandwidth) of 0.35. The kernel shows the (smoothed) path of the average. Most of the variation at the high end – notably the negative observations – is due to the oil-rich countries, so the 563 observations for OPEC members are singled out and omitted in the estimate of the kernel.[9]

Figure 2. The scatter of all N = 8,874 from 1950 to 2010

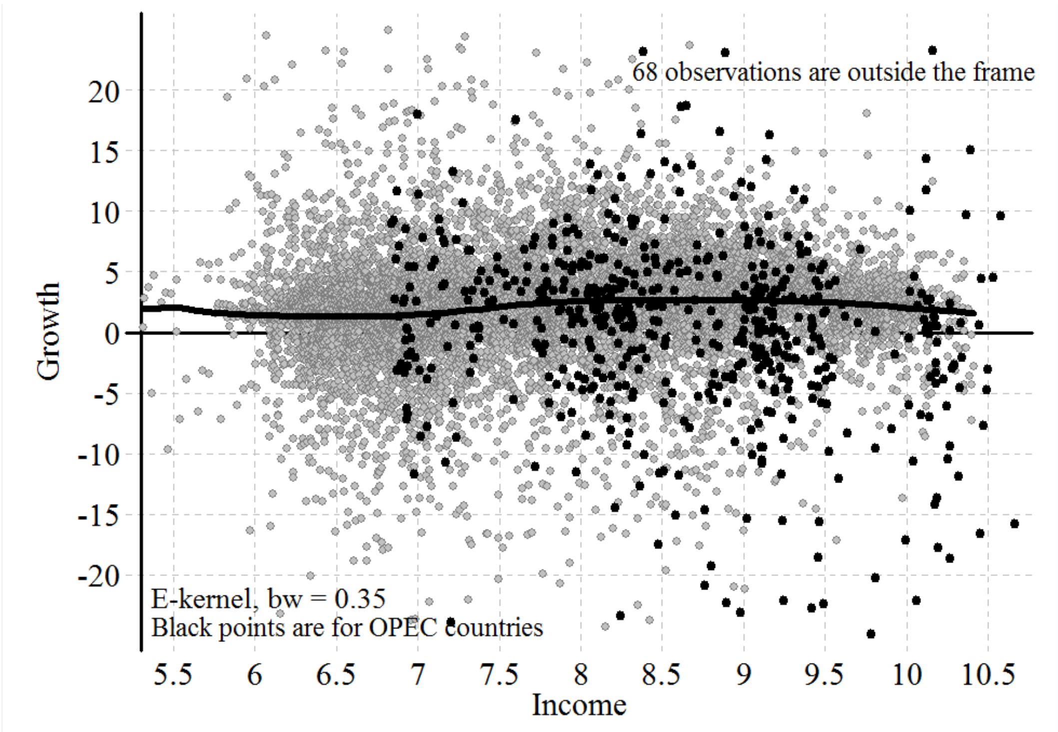
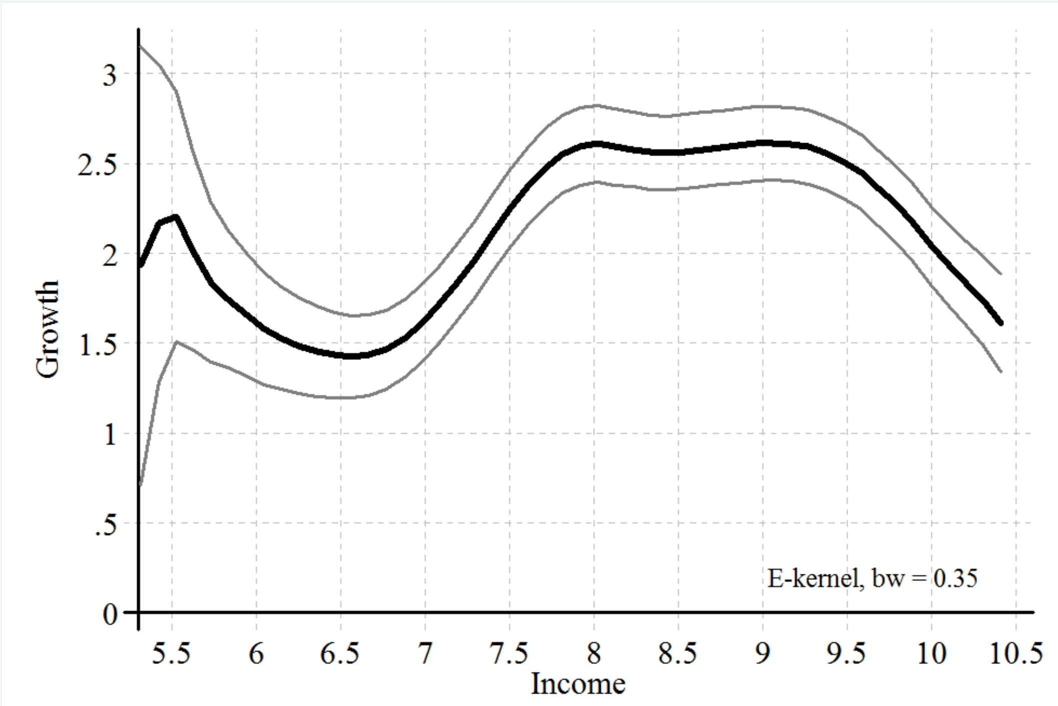



Figure 3 shows how the kernel curve looks when the scatter points are suppressed so that the vertical axis can be enlarged. The 95% confidence intervals are added. Figure 2 shows that the kernel is supported by few points at both ends, but it looks as if the scatter narrows at the high end when the OPEC countries are omitted.

9. The kernel for OPEC observations falls throughout and becomes negative from an income level of about 9. When the OPEC points are included in the estimate of the kernel, it is indistinguishable in most of the range, but it falls more at the high end and becomes negative, see Paldam (2015).

Figure 3. The kernel curve from Figure 2 enlarged




Note: The thick black curve is the kernel. The two gray curves give the 95% confidence interval. This is Figure 10b in Paldam (2015).

The basic form of the kernel-curve is robust to the bandwidth, to a break-up of the time period, and to averaging of the growth variable.[10] The key observation is that the confidence interval is so narrow around the average curve that the non-linearity of the curve is highly significant. There is no way a straight line can be drawn within the confidence interval.

The confidence interval is rather narrow except at the low end, where it widens as expected from Figure 2. This also means that it is possible to draw a horizontal line from $y$ = 5.5 to 7, so that the little top at 5.5 and the negative slope from 5.5 to 6.5 is insignificant. The fact that the confidence interval does not widen at the top suggests that the fall in density of the point is offset by a fall in its variation, as will be confirmed in section 4. The kernel-curve gives an answer to the two questions in the introduction:

A *low level equilibrium trap*: Here that key observation is that no negative section is found. There may be a convergence in the interval from 5.5 to 6.5, but it is a convergence to

10. Paldam (2015) documents the robustness in three ways: (i) The ($g_{ni}$, $y_{i(-)}$)-kernel is calculated for three values of $n$ = 1, 5 and 10, which looks similar. (ii) bw is varied from 0.2 to 0.7: With a lower bw the curve becomes more wobbly, but it keeps its basic form. For higher bws the 'muddle' at the start disappears and the peak becomes sharper. (iii) The kernel-curve is estimated for all decades separately: It looks much the same for 4 of the 6 decades, but in the 80s the debt crisis of the LICs and MICs gives many countries low growth, and in the 90s the kernel has an extra dip for the MICs due to the large group of countries going through the change from socialism to capitalism.

6.5, where growth becomes faster. It also means that the traditional steady state of a stable income is not visible in the data from 1950 onwards. Countries grows at all income levels.

*Convergence or divergence*: The kernel-curve has a significant hump in the middle. The countries diverge before the hump and converge after. It looks as if the hump has a rather flat top, which peaks somewhere between 7.8 and 9.5.

It is important that that the curve is positive throughout – the average of all points used to calculate Figure 3 is 2.05% p.a., and as can be seen, it is 1.6% at the two ends and approximately 2.6% at the flat top of the hump. So MICs grow by 1% more than the HICs. This closes the gap by 1 *lp* each century. The gap between the LICs and the HICs is 3.5 to 4 *lp*s. So here the catch-up needs 4-5 centuries in average.

## 4. A kernel for the standard deviations

To analyze the variation in the growth rate, we have calculated a *moving standard deviation* $s_{11}(g)_i$, which gives a $(y_{i(-1)}, s_{11}(g)_i)$-pair. The *s*-series is made and analyzed as follows:

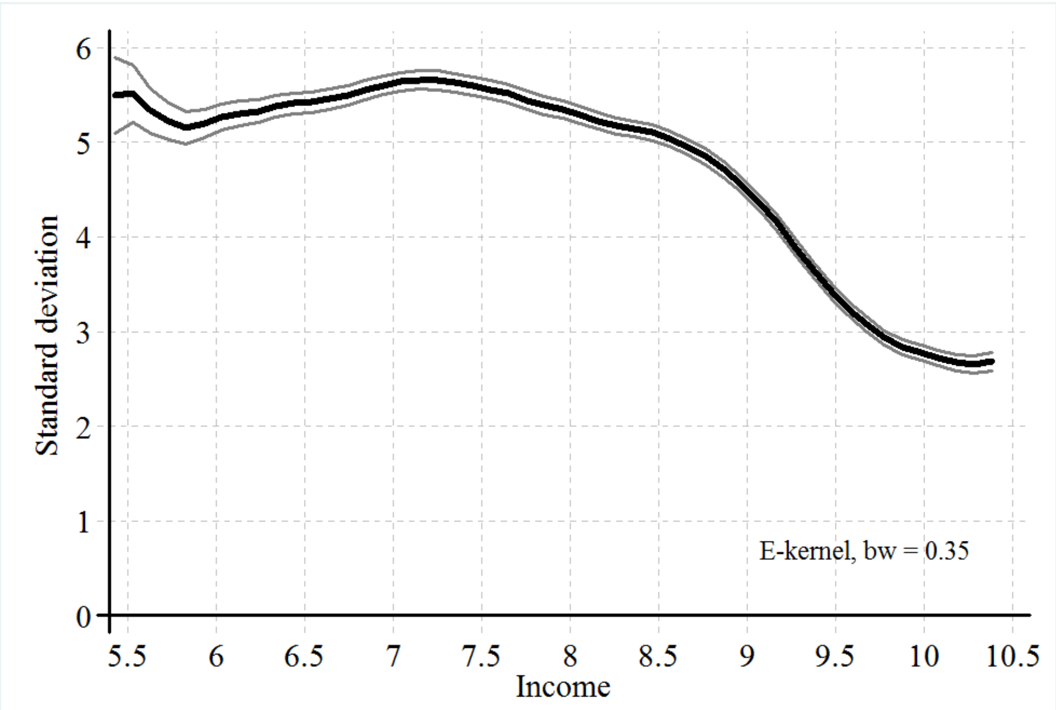
(1) First the $(y_{i(-1)}, g_i)$-series is sorted by $y_{i(-1)}$.

(2) Then the sorted $g_i$-series is replaced by $s_{11}(g_{i-5}, g_{i-4}, \ldots , g_i, \ldots , g_{i+5})$. Hereby the first 5 and the last 5 observations in the series are lost, so $N$ = 8,864.[11]

(3) Finally, the $(y_{i(-)}, s_{11i})$-series is analyzed by the same kernel-technique as before. The resulting kernel is termed a $\text{std}_{11}$-kernel.

Figure 4 shows that the $\text{std}_{11}$-kernel rises a little from the start till the peak in $y$ = 7.25. As y increases $s_{11}$ falls. In the end it has fallen to less than half of its peak value. This explains why the confidence intervals on Figure 3 do not fall at the high end, even when the observation density falls. Note also that the confidence interval falls and remains low throughout. The average of the 8,864 $s_{11}$s calculated is 5.01, as appears rather consistent with Figure 2.

Figure 4. The $\text{std}_{11}$-kernel on the same data as on Figures 2 and 3




11. The $s_{11i}$-series uses 11 observations for the calculations of the standard deviation. We have also made the calculations with 51 observations. It had no visible effects on the kernel shown. So the $\text{std}_n$-kernel is robust to choice of $n$ as long as $N$ is much larger than $n$.

## 8 Conclusions

The above analysis is done using a technique chosen to assume as little theory as possible. We hope the reader will agree that the analysis builds on very few assumptions. This will allow us to draw at least one rather strong conclusion about economic theory: It does appear that there is a transition in the growth rate: From moderate to higher and back again to moderate. Thus the one-sector steady state perspective on economic growth is problematic.

In addition our findings tell a story about the long-run dynamics of income of the world system of countries:

*Poor countries* (LICs) have a low and unstable growth. However, the growth is still at an average rate of 1.6% per year. The stagnating traditional economy is all but gone today.[12]

*Rich countries* (HICs) have a growth rate of 1.6% p.a. as well. The instability of this growth is half the one of the LICs.

*Middle-income countries* (MICs) have, in average, 1 percentage point higher growth than in the LICs and HICs. The peak is rather flat between $y$ = 7.7 and 9.5. During that period the variability of the growth rates falls.

An excess growth of 1 percentage point – as is found for the MICs – accumulates to one logarithmic point over a century. This means that many MICs will actually catch up with the HICs during the next century. It will take considerably longer for the LICs.

Due to the high variability of the growth of the LICs and MICs some countries will catch up much faster than others.

12. The world has about 10 countries with a *gdp* (GDP per capita) that is lower in 2010 than in 1950.

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## Appendix on the data definitions and the income scale

Both income and growth are calculated from the *gdp*, which is GDP per capita in real PPP prices. Income $y_{jt}$ = $\ln(gdp_{jt})$, where $j$ is a country and $t$ is time. *Growth*, $g_{jt} = 100(gdp_{jt} - gdp_{j(t-1)})/gdp_{j(t-1)}$. The data used for the calculations is stacked and sorted by income so that the country and time dimensions are scrambled and joined into one index $i$. We write ($y_{i(-1)}$, $g_i$) which are for the same country and two adjunct time periods ($t$ and $t$-1), however ($y_{i+1(-)}$, $g_{i+1}$) is unlikely to be from the same country as ($y_{i(-1)}$, $g_i$) or from the next time period. The *gdp* (GDP per capita) is in 1990 Geary-Khamis $. Income is the natural logarithm to *gdp*. The table shows the three closest matching countries to the income scale $y$ = 6, 6.5, ..., 10. Countries with an * miss the observation for 2010, so it has to be assessed. Nearly all countries of the west are between 10 and 10.3.

Table. Income scale: Closest matching countries in 2010

| Income | *gdp* | Match | Income | *gdp* | Match | Income | *gdp* | Match | Income | *gdp* | Match |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 400 | Burundi*<br>Niger<br>CAR* | 7 | 1100 | Mali<br>Korea N*<br>Kenya | 8 | 3000 | El Salvador*<br>Libya*<br>Philippines | 9 | 8100 | Costa Rica<br>China<br>Turkey |
| 6.5 | 670 | Guinea*<br>Madagascar<br>Haïti* | 7.5 | 1800 | Nicaragua*<br>Nigeria<br>Ghana | 8.5 | 5000 | Indonesia<br>Bahrain<br>Ecuador | 10 | 22000 | Korea S<br>Japan<br>Ireland |