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# A hump-shaped transitional growth path

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## Abstract:

Kernel regression estimates reveal a statistically significant hump-shaped relation between the growth rate and the level of income in cross-country panel data. This result survives a number of robustness tests, but it does not hold for the sample of OPEC countries. Interpreted as a transitional growth path, a hump-shaped relation between the growth rate and the level of income cannot be explained with the one-sector workhorse model of growth empirics. A two-sector model generates a hump-shaped transitional growth path that is similar to the one observed in the data.

Keywords: Kernel regressions, transitional growth path, two-sector model

JEL: O4, C4

*Conference version:* The data of the two authors are only coordinated to 95%. Thus small inconsistencies are still included – they will be weeded out in the next version.

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## 1. Introduction

Few relations in growth empirics have been studied as much as the one between the per capita real growth rate and the income level. We think that agreement has been reached on following points: An economy in steady state grows by the rate of technological progress. Two basic steady states exist: Countries in the *traditional* steady state have a stable technology giving a low stable income.<sup>3</sup> Countries in the *modern* steady state use a modern technology giving a much higher and growing income. Modern technology is international, so wealthy countries converge. The numbers of countries in the two steady states is slowly shifting so that the group of wealthy countries rises.

Most countries are in the *Grand Transition* from the traditional to the modern steady state and hence they are not in steady state.<sup>4</sup> When countries leave the traditional steady state growth must increase; and when countries catch up to become wealthy, they must have had a period of higher growth. A *growth-diagram* depicts the growth rate,  $g_{it}$ , as a function of initial income,  $y_{it-1}$ . It follows that the growth diagram is likely to be *hump-shaped*.

The paper generates growth-diagrams by kernel regressions on a large data set. The kernels work with data sorted by income, and other differences across countries and over time are effectively randomized. The size of the data set allows us to break up the sample in many dimensions in order to check the robustness of our main result.

The three main results are: (1) The growth diagram is indeed hump-shaped, and a linear path inconsistent with the data. (2) One exception is found in very resource rich countries, where a linear growth path and a global equilibrium occur. (3) The variation around the transitional growth path decreases at higher levels of income. Various robustness checks of our main results are included in a detailed online appendix to this paper.

Section 2 provides brief notes on selected contributions to the literature on transitional growth. This literature has mainly focused on methodical innovations for estimating the rate of *conditional* convergence rather than on estimating a common transitional growth path. Section 3 describes our data set. Section 4 reports our empirical results. Section 5 simulates the observed hump-shaped growth path using the two-sector model. Section 6 concludes.

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3. Several such technologies exist, so several traditional steady states exist. The key point is that the technology is stable. It seems that both China and most of Sub Sahara Africa had no growth from 1000 to 1700, but China had more advanced technologies and twice the GDP per capita.

4. The Grand Transition has other names such as the Modernization or the Industrial Revolution, but it is important that it is a transition and it is much more than industrialization.

## 2. Notes on the growth literature

These notes argue that the prodigious effort of growth research has an important skewness.<sup>5</sup> Researchers in economic growth know that most countries are in transition, but somehow it is assumed that this can be handled by refinements in the basic linear model, discussed in section 2.1. Section 2.2 surveys some of the empirical growth results. Section 2.3 discusses our use of kernel regressions, and how the kernels found should be interpreted.

### 2.1 *The workhorse model: is it the right tool?*

The basic linear workhorse model of growth empirics is derived from the Solow model (Solow 1956),<sup>6</sup> where  $y$  is income,  $e$  is the residuals, and  $t$  is time.

$$(1) \quad \ln y_t - \ln y_0 = f(\ln y_0) + e_t,$$

In panels where time is supplemented by countries  $i$ , equation (1) yields the following estimating equation:

$$(2) \quad g_{it} = \alpha_{[it]} + \beta y_{it-1} + [\gamma_1 z_{1it} + \dots + \gamma_n z_{nit}] + u_{it}, \text{ where } g \text{ is growth}$$

The two []-brackets contain ‘optional’ features. The first brackets show that the constant may be broken into time and country fixed effects. The second bracket holds specific controls – notably for country heterogeneity. The two brackets are difficult to apply at the same time.

Without the brackets, the estimate of  $\beta$  is a test for absolute convergence ( $\beta < 0$ ) or divergence ( $\beta > 0$ ). The typical result is insignificant divergence.

With one or two of the brackets included  $\beta$  tests for convergence conditional of the variables in the bracket. Already, fixed effects for countries  $\beta < 0$ , gives conditional convergence. If countries were sufficiently alike they would converge,<sup>7</sup> so growth miracles would happen at low levels of income and fade out subsequently.

As there is no absolute convergence, countries may have their own transitional growth path conditional on the determinants of the steady state (conditional  $\beta$ -convergence) – which implies that there is no cross-country pattern of long-run development, i.e., no common

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5. The four stately volumes of the handbook of economic growth have now reached almost 3,500 pages, with an index of authors of about 70 pages – our notes to the literature are notes indeed.

6. The theoretical foundation for equations (1) and (2) has been developed around 1990 (DeLong 1988, Barro 1991, Barro and Sala-i-Martin 1992 Mankiw *et al.* 1992).

7. Depending on the reasons given for the lack of convergence this may/may not be a tautology.

transitional growth path.<sup>8</sup> However, there is strong empirical evidence that a wide range of socio-economic have clear transitions paths. The variables have a different stable level at the two steady states and a clear transition between the two levels:

The birth rate and the mortality rate decline, so do the share (in GDP) of agriculture, religiosity and corruption, while the likelihood of democracy increases.<sup>9</sup> The details of the transitions and the causality from the level of income to the said variable can of course be debated, but their empirical relevance as a *general* pattern of development over long periods of time, say, the last two centuries, can hardly be disputed.

The transition pattern is non-linear. During the transition it may look linear, but it is horizontal at the two steady states and it has to bend to reach the horizontal ends. Given the transitions of all (?) socio-economic variables for a broad sample of countries, it is puzzling that there seems to be no systematic pattern for the (transitional) growth rate and the level of income for a broad sample of countries? Put differently, if growth empirics suggests that countries converge to individual steady states of per capita income, why don't we see individual steady states, e.g., for birth and mortality rates, the share of agriculture, religiosity, corruption, and democracy? Our hypothesis is that the log-linear transition path derived from the one-sector workhorse model of growth empirics could be a misleading starting point in the search for a cross-country pattern of long-run growth because it excludes the existence of a nonlinear transitional growth path by default.

## 2.2 *Some empirical findings*

Research in economic history and long-run development has claimed that two basic steady states exist: the traditional and the modern (Lewis 1954, Rostow 1960).<sup>10</sup> It led to the idea that they coexist during the transition as two sectors, where the modern sector gradually takes over (Ranis and Fei 1961). In the 1970s two-sector models were a key theme in development economics. Models with two sectors that are moving relatively is not, of course, in steady state. However, in the last 35 years this insight has been taken over by one-sector models, until Lucas (2009) updated the two-sector model.

The historical income data collection put together by Maddison (2001, 2003), has confirmed the existence of two steady states. The traditional (static) steady state is described

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8. The convergence literature is covered by a meta-analysis (Abreu *et al.* 2005).

9. The literature on socio-economic transitions is vast. Examples for a discussion of the transitions mentioned include (Galor 2005a, Gundlach and Paldam 2009a, Paldam and Gundlach 2013, 2017, Herrendorf *et al.* 2014).

10. Rostow (1960) proposed a form of the curve with distinct stages, however with a more realistic softening it does become a nice hump-shaped curve (Laursen and Paldam 1982).

by Malthus (1803). Neoclassical growth theory since the advent of the Solow model deals with fluctuations around the modern steady state and with the determinants of the steady state growth rate (Romer 1990). Unified growth theory (Galor 2005b) studies both the traditional and the modern steady state in one model.

The traditional steady state prevailed throughout most of the history of mankind. Average incomes were low and incremental technology advances supported a very slow but steady increase in population size. Larger populations meant more ideas. Hence population growth translated into a faster rate of technological change. The transition to the modern steady state with a persistent growth rate of per capita income in the range of 1-2% first took off in a few Western countries and spread in North and Central Europe their offshoots about two centuries ago. Over the last 50-100 years the transition spread to European periphery, and to East Asia, notably Japan and the four Asian Tigers (Hong Kong, Singapore, South Korea, and Taiwan) that have caught up with the high income countries,<sup>11</sup> sometimes due to growth miracles during the transition. These success stories hardly qualify as a *general* pattern of catching up and long-run development.

In contrast it has often been claimed that poverty traps keep countries in a traditional steady state (Aziarides and Stachurski 2005), or that the world income distribution is dominated by two rather isolated peaks, one for rich countries and the other for poor countries (Quah 1996). We find little support for these ideas.

One of the first empirical studies of  $\beta$ -convergence, using a version of equation (1) was Baumol's (1986) cross-country result of wealthy countries showing unconditional convergence. For a broad sample of countries, evidence for the "iron law" of a theoretically predicted convergence rate of about 2% has only been found by assuming that countries are converging to different steady states (Barro 2015). Empirical research on conditional convergence has no longer asked whether poor countries are catching up with rich countries. The focus has instead been on estimating the speed of the adjustment toward a country-specific steady state path and, not in estimating a common transitional growth path.

By controlling for country- and year-fixed effects in addition to steady state determinants like factor accumulation and population growth, Islam (1995) moved the discussion still further away from the initial research question on cross-country catching up and convergence.

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11. Various counts of HICs (high-income countries) exist. The OECD (Organization for Economic Co-operation and Development), a club of mostly rich countries, started in 1961 with 19 members; it has 35 members today. The World Bank designated 28 countries and 6 microstates as HICs in 1987; today the tally is 50 countries plus 29 microstates. Members of OPEC (Organization of the Petrol Exporting Countries) are a special case of HICs discussed in Section 4.3.

Moreover, the concept of convergence has been almost completely hollowed out by also allowing for country-specific rates of technical change (Lee *et al.* 1998). The problem is that with each "commonality" restriction eliminated from the data, the number of potential steady states will increase and may finally exceed the number of cross-section observations.<sup>12</sup> Hence conditional convergence has become a concept with little relevance in discussions of a potential pattern of long-run growth and development, which requires a common transitional growth path.

Apart from the fading conceptual basis for empirical studies of transitional growth, Monte Carlo simulations of growth regressions by Hauk and Wacziarg (2009) reveal the missing robustness of dynamic panel estimators like Difference-GMM (Caselli *et al.* 1996) and System-GMM (Bond *et al.* 2001) in the presence of multiple data problems. The claimed superior performance of a simple time averaged OLS (BE) estimator in terms of the average estimation bias has been questioned by Ditzgen and Gundlach (2016), who show that the favorable simulation results depend on the selected parameterization of the data generating process. Eberhardt and Teal (2011) criticize that the Monte Carlo simulations by Hauk and Wacziarg (2009) for ignoring technology heterogeneity, variable non-stationarity and cross-section correlation. Yet it has not been shown that more robust results emerge if these additional data problems are actually taken into account, neither for conditional nor for unconditional  $\beta$ -convergence.

Hence the present state of empirical research on transitional growth paths is rather unsatisfactory. Despite many methodical innovations over the last 20 years, robust estimates of the rate of *conditional* convergence are missing. More importantly, the concept of conditional convergence as such comes close to a tautology by predicting that countries would transition to the same income level if they were the same. The relevant empirical question is whether countries, despite being different, have enough commonality along some dimensions to allow for a common transitional growth path.<sup>13</sup>

Moreover, the long-run transition of an economy from the traditional to the modern steady state is probably not the same process as the adjustment of an economy that is close to but not in its dynamic steady state. The latter is explained by the Solow model. Therefore, it becomes questionable why one should impose the log-linear Solow restriction on the

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12. For instance, if there are two otherwise identical countries with different shares of investment in GDP, there are two possible steady states. If population growth also differs, there are four potential steady states. If technology differs as well, there are eight potential steady states, and so on.

13. As an aside, assuming at least some cross-country commonality appears to be the very reason for using panel data in the first place, but then the cross-section variation is usually eliminated with country-fixed effects.

specification when estimating the transitional growth path from one steady state to another. Not finding an unconditional log linear transitional growth path, as in the empirical convergence literature, may simply indicate that the Solow model is not an appropriate tool for studying the transition from the traditional to the modern steady state.

After the modern steady state took off in a few Western countries and their offshoots, large cross-country income gaps have accumulated over the last 200 years. Since the most advanced countries like the United States apparently keep growing at a long-run rate of a little less than 2%, higher transitional growth rates in follower countries are needed to narrow the income gap with the leader. Some examples for a successful catching up exist, as already mentioned. But why should transitional growth rates necessarily peak at the lowest income levels, as implied by the transitional dynamics of the Solow model? A low level of capital accumulation may indicate a high rate of return but it may also indicate the absence of property rights. For instance, per capita income growth in the early industrial economies took off near the end of the 19<sup>th</sup> century, when average incomes were already substantially higher than before the Grand Transition.

If growth miracles mainly occur after a certain income threshold has been reached, growth diagrams should show a hump-shaped transitional growth path.

### 2.3 *Growth diagrams from kernel regressions*

A kernel regression is a smoothed moving-average (MA) process with a fixed bandwidth, with two parameters, a smoothing expression (kernel) and a bandwidth (bw). The (stata) program used allows three choices: kernel, bandwidth and degree. Two defaults are used throughout. We use Epanechnikov's kernel for the smoothing, and the degree is zero.<sup>14</sup> However, we have experimented a great deal with the bandwidth as will be discussed.

The kernel technique has the big advantage that it does not impose a functional form on the estimation equation. This allows for a data-driven identification of a growth-income relation of any shape. Kernels come with confidence intervals that allow us to determine if various functional forms proposed by a theory are consistent with the data. It is inconsistent if a curve with the form proposed cannot be drawn within the confidence intervals. When the 95% intervals are narrow, the kernel is good representation of the data. If the intervals remain large even for high values of  $N$ , the countries cluster in groups that need to be identified. With the exception of the OPEC group (see section 4.3) no groups have been found.

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14. The results are robust to the choice of kernel for large data sets, and the degree is only relevant for polynomial smoothing.

Kernel regression has two main disadvantages: (i) The cross-section and the time dimensions of the data are merged, which assumes the equivalence hypothesis holds, so that the cross-country pattern is a good representation of the long run. (ii) It is a univariate technique, which only allows for a single explanatory variable. Both disadvantages matter less for large sample sizes.

Figure 1 summarizes our discussion of hypothetical transitional growth paths in the form of growth diagrams. Thus, the vertical  $g$ -axis gives the growth rate of per capita income. The horizontal  $y$ -axis gives the initial income (ln to GDP per capita).

Figure 1. Hypothetical growth diagrams  
 Figure 1a. Two linear transition curves

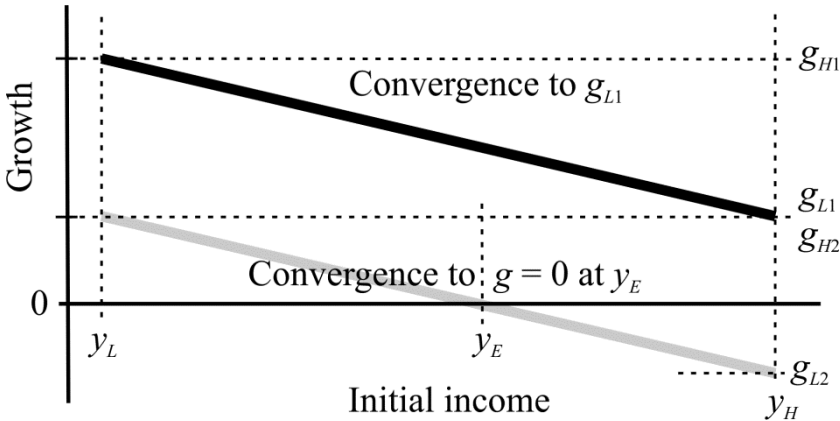
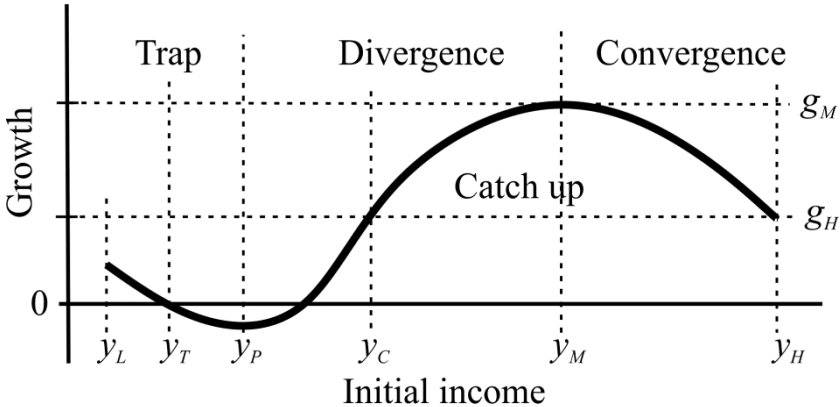


Figure 1b. A hump-shaped transition curve and a low-level equilibrium trap



Note: The figures are discussed in text



Figure 1a shows two log-linear relations predicted by equation (1), where the growth rate peaks at the lowest income level and falls subsequently.

The black curve has global convergence all the way from  $y_L$  to  $y_H$ , where growth falls from  $g_{H1}$  to  $g_{L1}$ . As  $g_{L1} > 0$  all countries converge to  $g_{L1}$ , which is the growth rate at  $y_H$ . Thus, the curve moves outwards. As the growth in  $y_L$  is higher than at  $y_H$ , the curve becomes shorter along the  $y$ -axis and presumably steeper. This does not look like facts we know.

The gray curve is the same as the black curve, but shifted downwards (so that  $g_{H2} = g_{L1}$ ). Hereby the curve comes to intersect with the  $y$ -axis, where growth is zero. This happens for  $y = y_E$ , that is an (global) equilibrium point to which all countries converge. The reader may wonder if this is ever observed. However, this case does appear in the OPEC group.

Figure 1b depicts a hump-shaped relation, where the growth rate peaks at an intermediate income level ( $y_M, g_M$ ). The peak divides the  $y$ -scale in a section to the right of  $y_M$  having convergence, and section to the left of  $y_M$  having divergence. The countries to the right of  $y_C$  has a higher growth  $g_H$  and thus they catch up with the rich countries at the top.

We have included a section from  $y_L$  to  $y_P$ , for low incomes where the curve has a negative slope. It is even drawn so that it intersects with the  $y$ -axis. Thus, a (local) equilibrium ( $y_T, 0$ ) occurs. It is the much discussed low-level equilibrium trap. If it exists, countries in the trap need a *big push* to pass the pivotal point  $y_P$  to escape.<sup>15</sup>

Thus, a great deal of the classical discussion of growth and development has an easily observable representation if the data can be summarized in the form of growth diagrams. This is where we turn to kernel regressions in section 4, but first we discuss the data and show a few conventional regressions.

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15. A big push has sometimes been tried when a strong state has enforce a dramatic increase in accumulation as in the Stalinist or Maoist models of industrialization, or it has been proposed as a recipe for development aid by Chenery and Strout (1966) or Sachs (2005).

### 3. Data, samples, and some linear regressions

Income is defined as the natural logarithm to  $gdp$  that is the GDP per capita, measured in real PPP prices. The growth rate is thus the growth of  $gdp$ .

The paper uses an updated version of the Maddison income data to estimate the relation between the growth rate and the level of income (Maddison 2001, 2003).<sup>16</sup> Annual data are available for some countries from 1800 to 2010, with gaps. One feature of the Maddison data is that they include observations for countries before they existed as independent political units and for ex-countries after their political dissolution.<sup>17</sup> To avoid overlapping observations, the series for ex-countries are discontinued when the series for the successor states begins.

After this adjustment, the Maddison sample covers  $N = 12,786$  data pairs for the growth rate from year  $t-1$  to year  $t$  and the level of income in year  $t-1$ . During the 19<sup>th</sup> century, about three quarters of the observations are from the early industrial countries of Western Europe and their overseas offshoots; then some Latin American countries join the sample but the data lacks representability before 1950. For each year from 1950 onwards, the sample holds at least 144 countries which together account for more than 95% of the world population. This subsample is labelled *All*, with  $N = 8,954$  (see Table 1).

Income is measured in 1990 international Geary-Khamis dollars. When converted to (natural) logarithms, the income data cover the interval from 5.3 to 10.5. A range of 5.2 logarithmic points (*lps*) represents a factor of 180 times of GDP per capita. To close such a gap within a century, one would have to have an average annual growth rate of 5.3 percentage points in excess of the growth rate of the rich countries. Comparable processes of catching up have happened, but only in a few cases and for shorter periods. Persistent excess growth rates in the range of 4-5% are rare. Hence, even under the most optimistic scenarios, the transition from the poorest to the richest countries is a question of centuries rather than decades.

The full data sample used in the paper (*All*) can be divided into the five standard regional groups West (high income developed countries), Latin America, Sub-Saharan Africa, MENA, and South and East Asia). The empirical results for the five regions do not differ

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16. Maddison (1926-2010) updated his data until 2008. Since then they have been updated as the Maddison Project (2013). For the underlying methodology and main results, see Bolt and van Zanden (2014).

17. For instance, observations for Sub-Saharan African countries start in 1950 when all but three countries were colonies. Observations for the successor states of Yugoslavia start in 1952 already; observations for the successor states of the USSR and Czechoslovakia start in 1990. At the same time, observations for Yugoslavia, the USSR, and Czechoslovakia run through 2010.

substantially from the results for the *All* sample although the income range differs across regions (see online appendix). However, there is one group of countries that displays a distinctly different transitional growth path, namely countries where the extraction of natural resources is the dominant industry.

All countries earn some resource rent. It is typically 1 to 2% of GDP only, but some countries have abundant resources and the resource rent becomes a major part of GDP. Such countries may reach high income levels exclusively from resource rent, without going through the systematic social, political, and cultural changes that define the transition from the traditional to the modern steady state. These countries often maintain a structure of society and its institutions that is typical for low income levels, with only gradual changes toward modernity.

The oil countries are the most extreme version of this case of (missing) socioeconomic development. The oil sector tends to be a small international enclave in the host country, employing few ‘natives’ and using a foreign technology. Such sectors are normally carefully fenced and heavily guarded so they are indeed an enclave. Their effect on the rest of the economy is driven by the tax on the resource rent that flows into the treasury. OPEC membership is used as a proxy to identify this special group of countries. That is, the *All* sample is divided in the *Main* group with  $N = 8,377$  observations and the *OPEC* group with  $N = 577$  observations.

Table 1. Descriptive statistics for alternative samples

	$N$	Countries	Average annual growth rate	Standard deviation
<i>All</i>	8,954	?	2.03	5.95
<i>Main</i>	8,377	163?	2.12	5.67
<i>OPEC</i>	577	?	0.84	9.04

Source: Maddison Project (2013), unbalanced cross-country panel data, 1950-2010. Own sample adjustments as explained in text.

In order to make sure that our *Main* sample does not differ systematically from samples that have been used in the convergence literature, we employ a most parsimonious empirical panel model to estimate the rates of unconditional and conditional  $\beta$ -convergence. The panel version of equation (1) reads

$$(2) \quad \ln y_{it} - \ln y_{it-\tau} = f(\ln y_{it-\tau}) + e_{it} \quad \text{with} \quad e_{it} = \mu_i + \eta_t + \varepsilon_{it}$$

where  $t$  denotes the end of a time period of duration  $\tau$  in country  $i$  and  $t-\tau$  denotes the beginning of that period. The error term  $e_{it}$  now includes country- and time-fixed effects in addition to the random factors  $\varepsilon_{it}$ . The inherent randomness of the growth rate is likely to increase with smaller  $\tau$ . Country-fixed effects ( $\mu_i$ ) allow for country-specific steady states; time-fixed effects ( $\eta_t$ ) allow common effects that are assumed to be the same for all countries.

A linear regression specification of equation (2) is given by

$$(3) \quad \ln y_{it} - \ln y_{it-\tau} = c + \beta(\ln y_{it-\tau}) + \mu_i + \eta_t + u_{it},$$

where  $c$  is a regression constant,  $u$  is assumed to be a random error term,  $\beta$  is the parameter of interest. An estimate of  $\beta < 0$  indicates a log-linear transitional growth path to income convergence (here conditional on the two fixed effects). It can be shown (Mankiw *et al.* 1992) that the rate of convergence  $\lambda$  can be derived from the estimate of  $\beta$  as

$$(4) \quad \beta = -(1 - e^{-\lambda\tau}) \Rightarrow \lambda = -\ln(1 + \beta) / \tau,$$

where the lengths of the time period is usually set to  $\tau = 5$  in panel studies.

Without the inclusion of the two fixed effects, the standard result reported in the literature is that  $\beta > 0$  but statistically insignificant, so convergence is rejected. Once fixed effects and additional explanatory variables are included, it has usually been reported that estimates of  $\beta$  turn negative and imply rates of convergence between 1% and 4%, depending on the selected samples and specifications (Islam 1995).

Table 2. Explaining growth by income and fixed effects for all data

Reg.	Income	Constant	Fixed Effects	R <sup>2</sup>	N
(1)	0.135 (2.4)	0.965 (2.1)	none	0.0006	8,954
(2)	-0.846 (7.3)	9.992 (9.2)	countries	0.0621	8,954
(3)	0.252 (4.4)	0.953 (1.7)	time	0.0300	8,954
(4)	-0.614 (4.6)	8.977 (6.9)	both	0.1311	8,954

Note: Estimates based on unbalanced panel data, 1950-2010; t-ratios in parentheses. The constant in the FE-estimates is reached by deleting The 29 of the 163 countries, where  $-0.2 < t < 0.2$ .

Our sample replicates these results. With  $\tau = 1$  as used with the Kernel regressions in the next section, we find that  $\beta > 0$  without the inclusion of fixed effects (Table 2, first row). This suggests the absence of a log-linear common transitional growth path as imposed on the data by equation (3). Once fixed effects are included the estimate of  $\beta$  turns negative, which implies the presence of a log-linear conditional transitional growth path and convergence rates between ?% and ?% (to be shown). Changing the length of the period to  $\tau = 5$  does not change the result (to be shown). But as discussed in Section 2, conditional convergence rates tell us nothing about a common long-run transitional growth path.

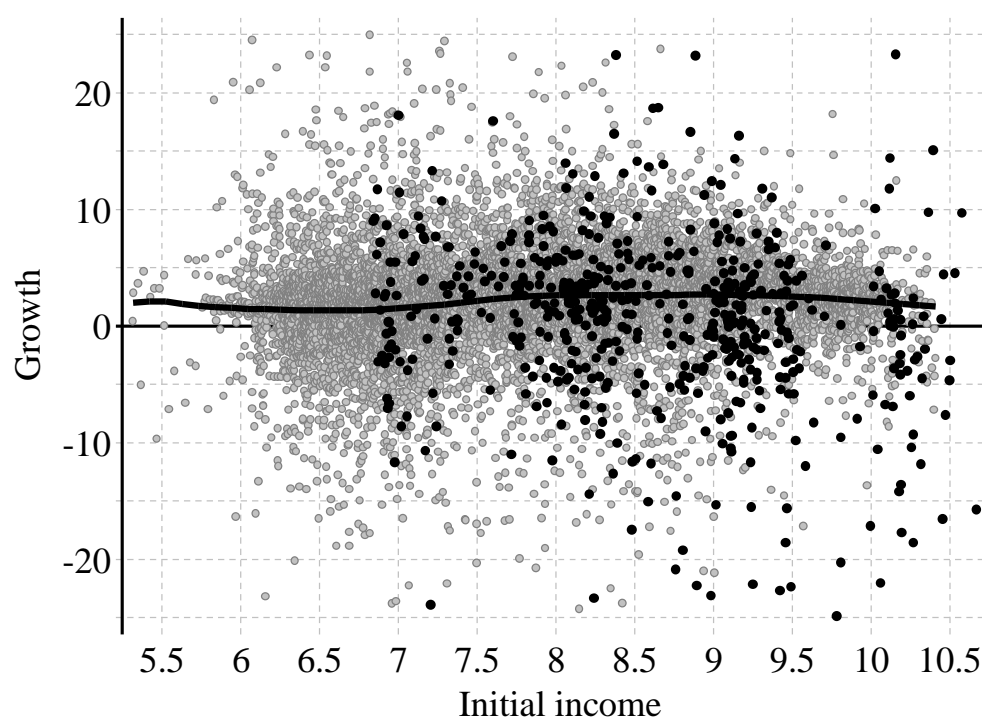
## 4. Kernel regression estimates of the transitional growth path

Section 4.1 gives the scatter graph of the data. Sections 4.2 and 4.3 report Kernel estimates of the transitional growth path for the *Main* and the *OPEC* samples. Section 4.4 looks at changes of the variation around the estimated transitional growth path with rising levels of income.

### 4.1 A scatter graph of the 8,872 data points

Figure 2 shows the (missing) correlation between the annual growth rate and the level of per capita income at the beginning of each year (initial income) for the *All* sample. Since income is scaled in natural logarithms, an increase by one log point represents an increase of income by a factor of 2.72. Hence the income range of 5 log points in Figure 2 implies that the income of the poorest and the richest sample countries differ by a factor of 148.

Figure 2. The annual growth rate vs. initial income (*All* sample)



Note: Cross-country panel data ( $N=8,874$ ), 68 observations are outside of the frame. The *OPEC* sample is indicated by black dots. The bandwidth is 0.35 and the kernel excludes *OPEC* sample.

The wild scatter of the data points means that any common transitional growth path will at best explain a modest part of the variation in growth rates. This can be no surprise: long-run

transitions will be affected by many variables and of course also by random events, not only by initial income. A log-linear transitional growth path can be ruled out according to the regression results in Table 2, but a wild scatter as such does not preclude a non-linear relation between the growth rate and the level of income.

The black line shown in Figure 2 is the kernel estimated with a bandwidth (bw) of 0.35. Most of the variation of the scatter points at the high end – notably the negative observations – is due to the OPEC members (in black), which have been omitted from the estimation of the reported kernel regression line.<sup>18</sup> It is difficult to see the kernel due to the wild scatter of observations.

The process of transition that is covered by the income range in Figure 3 often includes shifts in the income and power distribution among the major groups of the society, such that old political alliances are likely to break down and new ones will form. Hence it can be no surprise that the long-run transitional growth path identified in Figure 3 is overlaid with all sorts of unstable growth periods that account for the noise in the data shown in Figure 2.

#### 4.2 *The hump-shaped transitional growth path for the Main sample*

Figure 3 shows how the kernel regression when the scatter points are suppressed so that the vertical axis can be enlarged. The first finding is that no straight line can be drawn within the confidence interval. Thus, equation (1) is rejected by the data.

In contrast, the data between  $y_P$  and  $y_H$  support the hump-shaped transition curve from Figure 1b.<sup>19</sup> The only deviation from Figure 1b is that the peak is rather flat in the interval from  $y = 7$  to 8; but a nicely rounded ‘hump’ with a peak around  $y_M$  can be drawn within the confidence interval. This form is taken to be the stylized form of the transition curve.

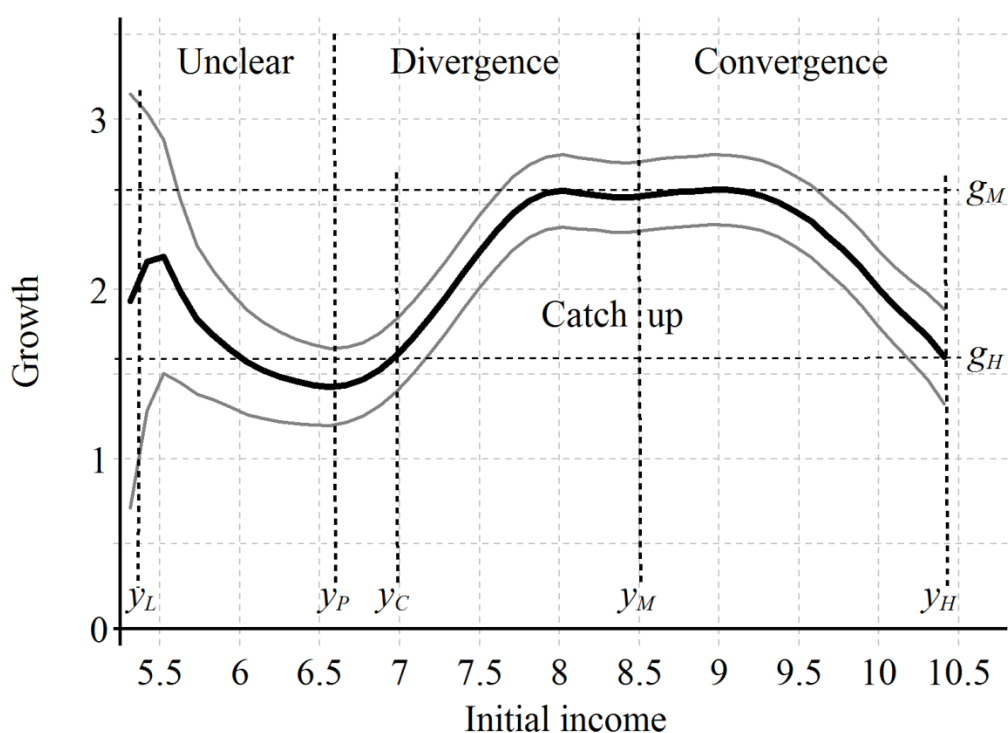
At low income levels between  $y_L$  and  $y_P$  the kernel does have a negative slope, but it is supported by rather few data points so the confidence intervals widens to make the negative slope is insignificant. It is possible to draw a horizontal line from  $y_L$  to  $y_C$  within the confidence interval. Also, the estimated Kernel line is significantly positive over the complete income range. This finding excludes the existence of a poverty trap.

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18. See section 4.3 on the Kernel regression line for the OPEC members.

19. The online appendix reports estimates of the Kernel regression line for alternative country groups and time periods for a range of bandwidths from 0.2 to 0.7. If the OPEC members are included in the sample (*All* sample), the kernel line is indistinguishable from the line in Figure 3 over most of the range, but it falls more at the high end and reaches negative growth rates. Overall, the hump-shaped form looks to be rather robust in terms of the estimated narrow confidence intervals, which exclude alternatively shaped transitional growth paths in almost all cases considered. It should be noted that goodness of fit is not an issue with Kernel regressions, in contrast to standard regression estimates.

Figure 3. The growth diagram for the Main group



Note: Cross-country panel data ( $N=8,377$ ). The same as Figure 2, but the scatter is deleted, and the growth-axes is enlarged. The two (thin) gray lines show the 95% confidence interval.

Leaving aside the noise in the data, the stylized growth performance of a representative country varies with rising levels of income along the estimated Kernel line. For instance, a rich country at the right end of the Kernel line grows with about 1.6% per year, as indicated by the dotted horizontal line  $g_H$ . The vertical line  $y_C$  marks the intersection of the Kernel line with horizontal  $g_H$ -line. Countries with an income level that is below  $y_C$  grow slower than the richest countries. But since these countries also have a positive growth rate, they will eventually cross  $y_C$  and start to catch up with the richest countries.

Countries with income levels to the right of  $y_C$  are growing faster than the richest countries, so they are catching up. This seems rather positive, but the implied catch-up times are long because the growth differential equals one percentage point at best. For instance, a poor country starting with an income level of 7 log points (about int-\$ 1,100) has a relative income of 3% compared to the richest country with an income level of 10.5 log points (about int-\$ 36,300). If the poor country grows on average by one percentage point faster than the richest country, it would need about 280 years to reduce its income gap from 97% to 50%.

However, the estimated Kernel curve implies that such a scenario is even too optimistic, because only the flat top of the hump of the Kernel line predicts a transitional



growth rate of about 2.6%. E.g., if the poor country grows by only 0.7 percentage points faster than the rich country, it would take about 400 years to reduce its income gap to 50%. Moreover, the positive slope of the Kernel line between an income level of about 6.5 log points and the maximum of the hump implies that income levels will first diverge across countries before they start to converge along the negative slope of the Kernel line.

Dividing the income range at  $y_M \approx 8.5$  log points, i.e., at about the middle of the flat top of the hump, gives statistically significant standard regression estimates of divergence and convergence, but of course both estimates explain very little of the overall variation in the growth rates. Nevertheless, these results also suggest that any model of long-run transitional growth should be able to generate a non-linear transitional growth path.

Table 3. Linear regressions explaining the growth rate, constrained by level of income

Country group	Figure	Income	Constant	R <sup>2</sup>	N
		Divergence for income below hump			
Main group $6 < y \leq 8$	4	<b>0.855</b> (5.0)	<b>-4.176</b> (-3.5)	0.005	4,624
		Convergence for income above hump			
Main group $y > 8.5$	4	<b>-0.470</b> (-2.5)	<b>6.696</b> (3.8)	0.002	2,483
OPEC group (see Section 4.3)	5	Convergence for all observations			
		<b>-1.89</b> (-4.7)	<b>17.12</b> (4.9)	0.035	577

Note: See Table 1.

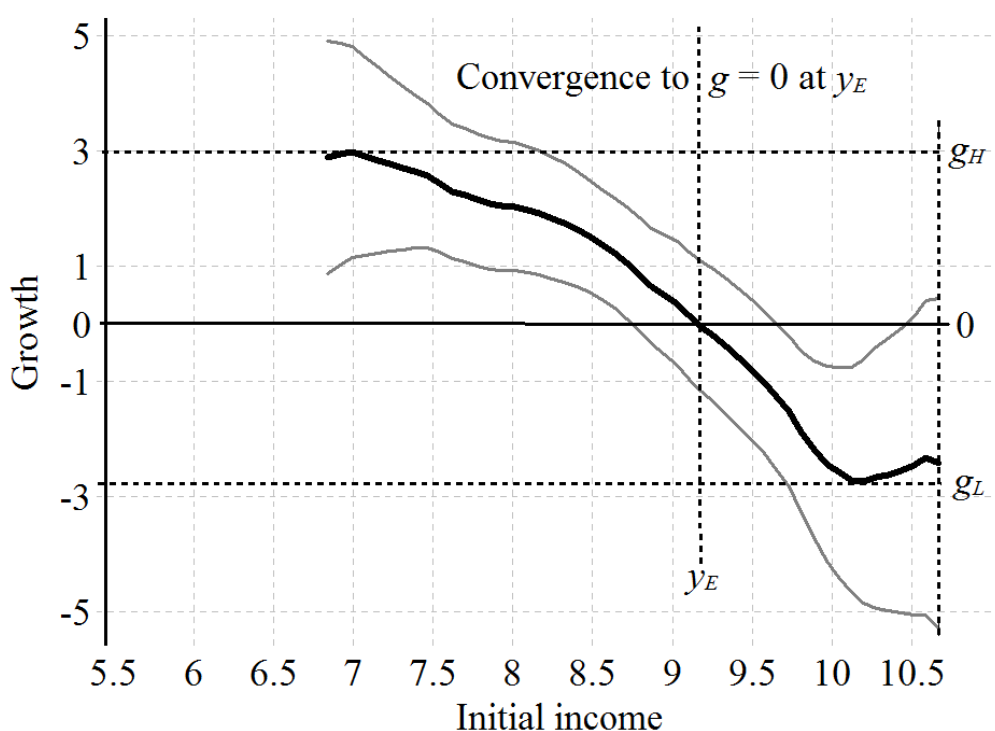
#### 4.3 *The unique transitional growth path of the OPEC countries*

Figure 4 shows the Kernel regression line for the *OPEC* sample. The horizontal axis employs the same income scale as in Figures 2 and 3, but here the kernel is estimated on 577 observations. Hence, the confidence interval and also the vertical axis are wider.

The path is consistent with a linear curve as drawn by the gray line on Figure 1a. It intersects the horizontal axis at  $y_E$  that is a global equilibrium.

As shown in the online appendix, the negative slope of the *OPEC*-Kernel is robust to a wide range of bandwidths between 0.1 and 1 and the estimated intersection point moves little for bandwidths between 0.1 and 0.7. A standard regression estimate shows a negative coefficient that implies a rate of convergence of about 2%, though statistical significance is somewhat below the 5% level (see Table 3).

Figure 4. The growth diagram the OPEC group



Note: See note to Figure 3, drawn with same y-axis.  $N = 577$ ,  $bw = 0.35$ .

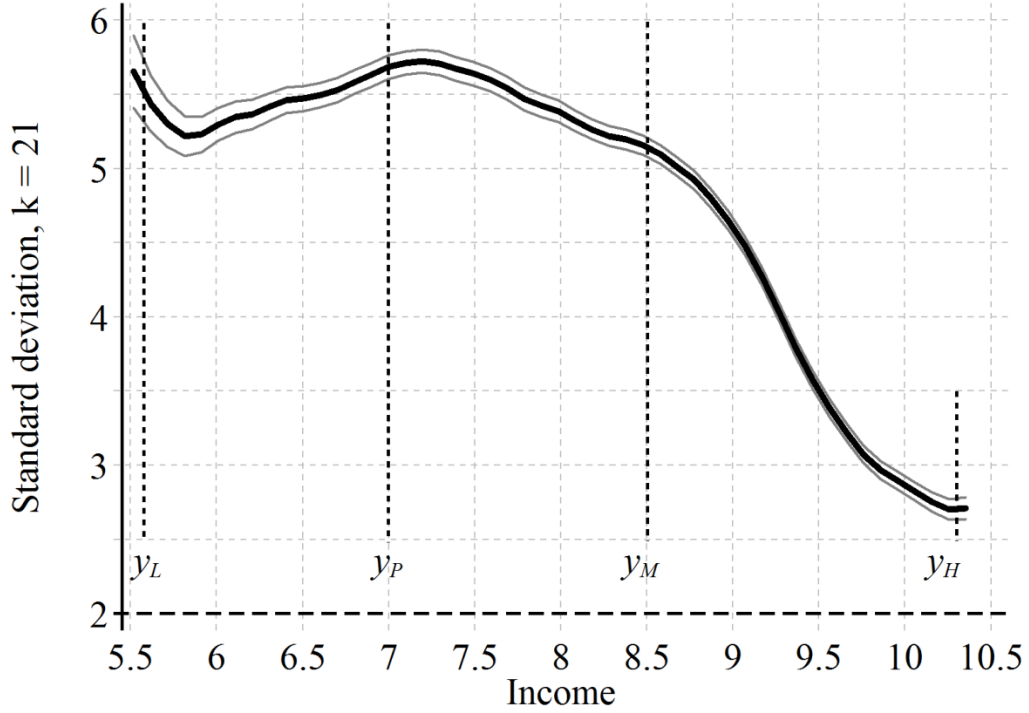
Taken at face value, Figure 4 suggests that a representative resource-rich country does not experience a transition to a modern dynamic steady state, but instead gets trapped at a per capita income in the vicinity of int-\$ 10,000. These countries may quickly reach relatively high levels of income, but then they tend to experience no further long-run growth.

This observation can be partly explained by the currency appreciation caused by the resource boom known as Dutch Disease (surveyed in Paldam 2013). The Dutch Disease model does predict a slowing down of growth, but it does not predict a global steady state equilibrium income, such as  $y_E$ . But perhaps the amazingly constant income of Venezuela over the last 60 years (even disregarding the present crisis) can be taken as a supporting case – especially as it happens at the income level predicted by the Kernel line in Figure 4.

#### 4.4 The increasing robustness of the transitional growth path with rising levels of income

As already mentioned above, we also find that the variation around the transitional growth path declines at higher levels of income. This is shown by calculating a *moving standard deviation* over a certain range of the income-sorted growth rates, which proceeds as follows.

Figure 5. The  $std_{21}$ -kernel on the same data as Figure 3



Note: Same data as Figure 3. The two incomes  $y_P$  and  $y_M$  are also from Figure 3.

The annual growth rate of country  $i$  is defined by  $g_{it} \equiv \ln y_{it} - \ln y_{it-1}$ . Sorting the pooled cross-country data by  $\ln y_{it-1}$ , the data pairs  $[g_{it}, \ln y_{it-1}]$  have been used to estimate the Kernel regressions of Figures 2-4. The income sorted data pairs are given by  $(g_j, \ln y_j)$  with  $j = 1, \dots, N$ , so a certain sequence of adjacent observations of the series  $g_j$  can be used to calculate a moving standard deviation.

A sequence is defined as  $S_n \equiv (g_{j-\kappa}, g_{j-\kappa+1}, \dots, g_j, \dots, g_{j+\kappa-1}, g_{j+\kappa})$ , where  $k = 2\kappa + 1$  equals the selected size of the window for calculating the moving standard deviation of the growth rate  $std(g_j^n)$ . This gives the data pairs  $[std(g_j^n), \ln y_j]$ , which can be used to estimate a Kernel regression line of the (moving) standard deviation with sample size  $N - 2\kappa$ .

The selection of the size of the moving window is arbitrary, so we have used alternative values of  $k$ . The shape of the estimated Kernel line is not affected by these alternative values as long as  $N$  is much larger than  $n$  (see online appendix). The Kernel line shown in Figure 5 is based on a moving window with  $k = 21$  observations.

Figure 5 shows that the kernel line of the standard deviation  $std(g_j^{21})$  rises a little

from the start till its peak at about 7.25 log income points. For higher income levels, the Kernel line displays a negative slope. The standard deviation of the growth rate falls to less than half of its peak value and the confidence interval around the Kernel line remains low throughout. The falling standard deviation of the growth rate explains why the confidence intervals shown in Figure 3 do not increase at high income levels when the number of observations falls in the *Main* sample. The average standard deviation of the growth rate across the whole income range is 5.06 ( $N = 8,357$ ), which reflects the narrow confidence interval over most of the income range in Figure 3.

## 5. Understanding the hump-shaped transitional growth path

Having identified a hump-shaped relation between the growth rate and the level of income that cannot be explained by the workhorse model of growth empirics, the question arises which other model can generate a hump-shaped transitional growth path and if so, which parameter constellations will simulate a transitional growth path like the one observed in the data. Since the one-sector does not work, a two-sector model of development introduced in section 2.2 is an obvious choice.

### 5.1 The two sector model of development

We shall use the new version of the model by Lucas (2009) as our point of departure.<sup>20</sup> This model has a large agricultural sector in combination with international and domestic spillovers (externalities) that delays the instant growth miracle that would otherwise emerge from the advantage of backwardness, at least according to standard neoclassical reasoning. We use this model for our simulations in the next section but calibrate it differently in order to generate the hump-shaped growth pattern reported in Section 4, which peaks at a rate of about 2.6% over an income range of int-\$ 3000 - 8000.

In the model, all countries are represented as one-factor "AK" economies (Rebelo 1991), where GDP per person is proportional to the single factor input. The labeling of the single factor input is not essential. As used below,  $K$  is labeled *capital* and may be interpreted as a broad concept of knowledge capital that also includes human capital.

Output and factor input of the leading economy are denoted in capital letters, such that

$$(1) \quad Y = B K ,$$

where  $Y$  is GDP (income) per person,  $K$  is capital per person, and  $B$  is a constant. In the leading economy, the stock of capital and per capita income are assumed to grow at the exogenous steady state growth rate  $\gamma$ , which is given by

$$(2) \quad K(t) = K(0)e^{\gamma t} \quad \Rightarrow \quad \gamma = \dot{K} / K = g_Y .$$

In any other (follower) economy, the stock of capital is assumed to evolve according to

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20. This section closely follows Lucas (2009, section III).

$$(3) \quad dk / dt = \dot{k} = \gamma k^{1-\psi} K^\psi ,$$

with  $\psi$  as an externality linked to the accumulation of capital.<sup>21</sup> Equation (3) implies that the growth equation of a follower economy is given by

$$(4) \quad g_y = \dot{k} / k = \gamma (K / k)^\psi .$$

Hence the growth rate of any follower economy positively depends on the proportional income distance to the leading economy ( $K / k$ ) and on the unknown size of the capital gap externality  $\psi$ . For any positive value of  $\psi$ , equation (4) implies a log-linear relation between the growth rate and the initial capital (respectively income) of a follower economy: a larger capital gap relative to the leader implies a higher growth rate of the follower, thereby implying a process of unconditional catching up that is not supported by the data discussed in Section 3.

Lucas argues that a large agricultural sector may act as a brake on catch-up growth because poor, largely illiterate, feudal societies at early stages of development may not be able to implement technology that is available from more advanced countries, at least not as long as a critical amount of capital has not been accumulated. He formalizes this hypothesis with a model that has "city" ( $c$ ) and "farm" ( $f$ ) as two sectors, which both produce a single output good that adds up to GDP.

Since cities are held to be the centers of intellectual exchange, they may generate positive externalities in the sense that the growth rate of a follower economy is affected by an agglomeration externality and by a sectoral productivity externality. City output  $y_c$  is produced with capital according to

$$(5) \quad y_c = k(1-x) ,$$

where  $x$  is the fraction of the labor force that is employed in agriculture, so  $x < 1$ . The *agglomeration externality*  $\zeta$  enters the growth equation (4) as

$$(6) \quad g_y = \gamma (1-x)^\zeta (K / k)^\psi ,$$

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21. Lucas interprets  $\psi$  as an index of openness to trade and technology inflows, where  $\psi = 0$  represents a completely closed economy and  $\psi = 1$  represents a completely open economy. Gundlach and Svendsen (2016) use the same model but interpret  $\psi$  as an index of social trust.

with  $\zeta > 0$  as a parameter of unknown size.

The *sectoral productivity externality*  $\xi$  enters the growth equation (6) via the production function of the farm sector, which is given by

$$(7) \quad y_f = Ak^\xi x^\alpha,$$

where  $y_f$  is farm output,  $A$  is a constant that includes farm land per person,  $\alpha$  is labor's share in farm production, and  $\xi > 0$  is a parameter of unknown size. Solving equation (7) for the employment share of the farm sector,

$$(8) \quad x = (\alpha A / k^{1-\xi})^{1/(1-\alpha)}$$

and inserting equation (8) into equation (6) gives the augmented growth equation of a follower economy as

$$(9) \quad g_y = \gamma \left[ 1 - (\alpha A / k^{1-\xi})^{1/(1-\alpha)} \right]^\zeta (K/k)^\psi.$$

The growth path described by equation (9) exhibits a number of distinct features. For instance, a positive growth rate can only result if the term in brackets (the added agglomeration externality) is positive, which is guaranteed by assuming a minimum amount of  $k$  that is large enough to allow for any growth at all. If  $k$  remains below the critical level, the predicted growth rate approaches 0 with  $k \rightarrow 0$ , all else constant. But if  $k$  is increasing beyond the critical level, the growth rate will reach a maximum where the growth drag of a large farm sector is overcompensated by the capital gap. With  $k \rightarrow \infty$ , the term in brackets and the capital gap term (in parentheses) will both approach 1, which implies that the growth rate of the follower economy ( $g_y$ ) will gradually approach the growth rate of the leading economy ( $\gamma$ ).

Hence equation (9) predicts a hump-shaped transitional growth path: a low growth rate at low levels of capital, a rising growth rate after a critical level of capital has been reached, and a gradually declining growth rate over the subsequent process of catching up with the leading economy in the long-run. The growth advantage of extreme poverty implied by the one-sector model of equation (4) is eliminated by the introduction of a second sector that

generates further externalities. What remains to be seen is whether reasonable parameterizations of equation (9) can actually generate a hump-shaped transitional growth path that resembles the hump-shaped pattern identified by the Kernel regression of Section 4.

### 5.2 *Simulating hump-shaped transitional growth*

Simulating the growth path of an initially poor hypothetical economy on the basis of equation (9) requires initial values for per capita income in the leading ( $Y_0$ ) and the following economy ( $y_0$ ) and also for the employment share of agriculture ( $x_0$ ) together with parameter values for  $\gamma$ ,  $\psi$ ,  $\xi$ ,  $\zeta$ ,  $\alpha$ , and  $A$ .

The initial level of per capita income of the leading economy is set to  $Y_0 = 12,000$  (int-\$), which equals about the income level of the United States in 1960. The initial level of per capita income of the following economy is set to  $y_0 = 1,000$  (int-\$), which is close to the average income level of poor though not extremely poor economies in 1960. Hence the initial income gap between the leading and the following economies is set to 8.3%. The constant growth rate of the leading economy is set to 1.8%, which reflects the average annual growth rate of per capita income of the United States in 1913-2010 (Maddison Project 2013).

The other parameter values are more difficult to motivate. Lucas employs equation (8) to describe the correlation between the employment share of agriculture ( $x$ ) and log GDP per capita ( $y$ ) in cross-country and time series data and finds a best fit by setting  $\xi = 0.75$ ,  $\alpha = 0.6$ . For given  $\xi$  and  $\alpha$ , the implied parameterization of  $A$  can be derived for a given value of  $k_0$ , which itself can be derived from the equations of the model for an initial income level  $y_0$ . With  $y_0 = 1,000$  (int.\$) as noted above, we derive parameterizations of  $k_0 = 714$  (int-\$) and  $A = 5.06$ .<sup>22</sup>

The parameterization of the capital gap externality ( $\psi$ ) is based on a theoretical range [0,1], where a value of 0 would apply for an economy that is completely closed to any spillovers from the leading economy, say, in the form of imports of advanced capital goods and technology, while a value of 1 would apply for an economy that can fully capitalize on the gains from backwardness, as assumed by the textbook Solow model. Real economies probably do not fall into these theoretical border cases, so we limit the range of parameterizations to [0.2, 0.4, 0.6], with  $\psi = 0.4$  as our preferred parameterization for the average

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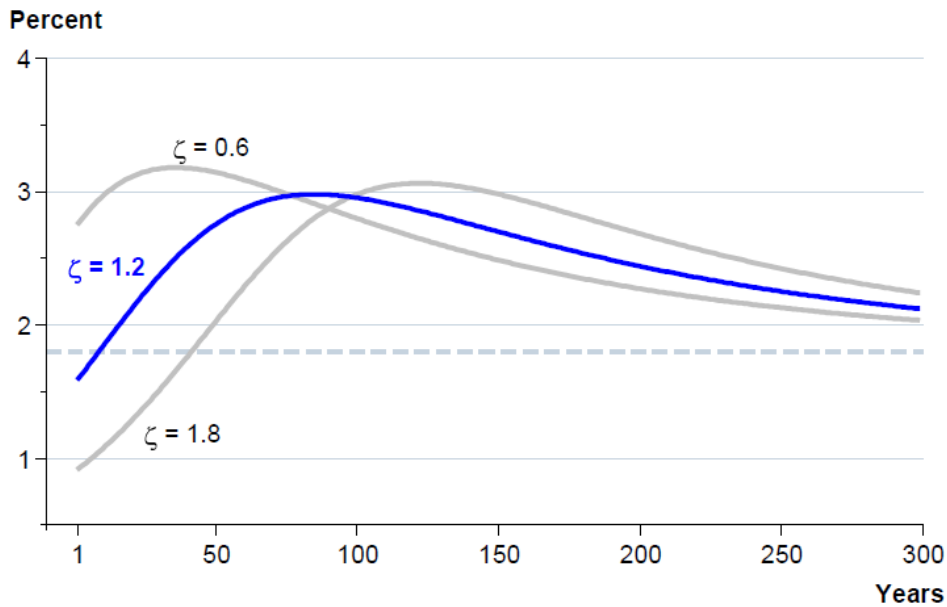
22. Detailed results are available upon request; see also the calculations in the appendix of Gundlach and Svendsen (2016).



sample economy in 1960.

These settings leave the agglomeration externality,  $\zeta$ , as the only unknown parameter.<sup>23</sup> Since empirical evidence on the size of  $\zeta$  is missing, we employ the alternative parameterizations [0.6, 1.2, 1.8]<sup>24</sup> in order to match the hypothetical growth path with the empirical hump-shaped growth path shown in Figure 4.

Figure 6. Simulated growth paths for different values of the agglomeration externality



Notes: Growth rate of hypothetical economy over time. Other parameters:  $\psi = 0.4$ ,  $\xi = 0.8$ ,  $\alpha = 0.6$ . Initial conditions:  $y(0) = 1,000$ ,  $x(0) = 0.6$ .

Figure 6 shows simulated growth paths of a hypothetical poor economy with an initial employment share of agriculture of  $x_0 = 60\%$  and a capital gap externality of 0.4.<sup>25</sup> The different paths represent the alternative parameterizations of  $\zeta$  for a given set of the other parameters. A hump-shaped growth path emerges for all three cases considered, but it looks that parameterizations with  $\zeta < 1.2$  tend to generate a path that is close to the log-linear path predicted by the one-sector model while parameterizations with  $\zeta > 1.2$  tend to generate delayed growth miracles that are also not supported by the empirical record. For  $\zeta = 1.2$ , the simulated growth path looks similar to the observed growth path. With an initial income level

23. For  $\zeta = 0$ , equation (9) equals equation (4).

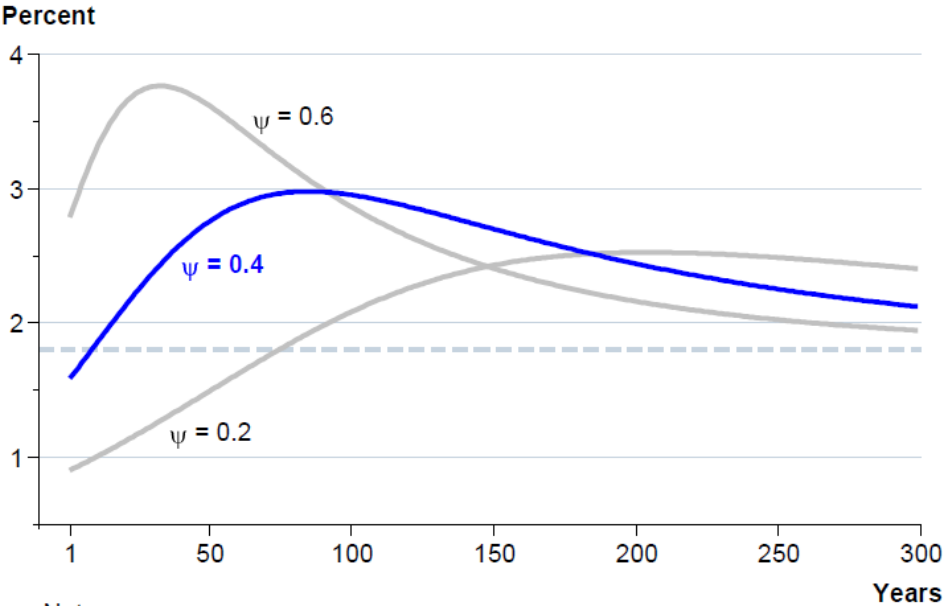
24. Lucas uses  $\zeta = 1$  as the preferred parameterization.

25. The simulation results are generated with Stata. The code is available upon request.

of \$ 1,000, the simulated growth rate starts below 2% but rises steadily until it peaks close to a rate of 3% after about 85 years, when the accumulated income level of the hypothetical economy has reached about int-\$ 8000. Subsequently, the growth rate gradually falls until it approaches the long-run steady state growth rate of 1.8%.

Independent from any specific parameterization, the underlying model always generates a growth miracle and income convergence in the very long run. But alternative parameterizations determine the timing and the size of the simulated growth miracles, which can be compared to observed growth miracles. Figure 7 provides a robustness check in the form of a variation of the capital gap externality ( $\psi$ ) by holding constant the value of the agglomeration externality at the preferred value of  $\zeta = 1.2$ .

Figure 7. Simulated growth paths for different values of the capital gap externality



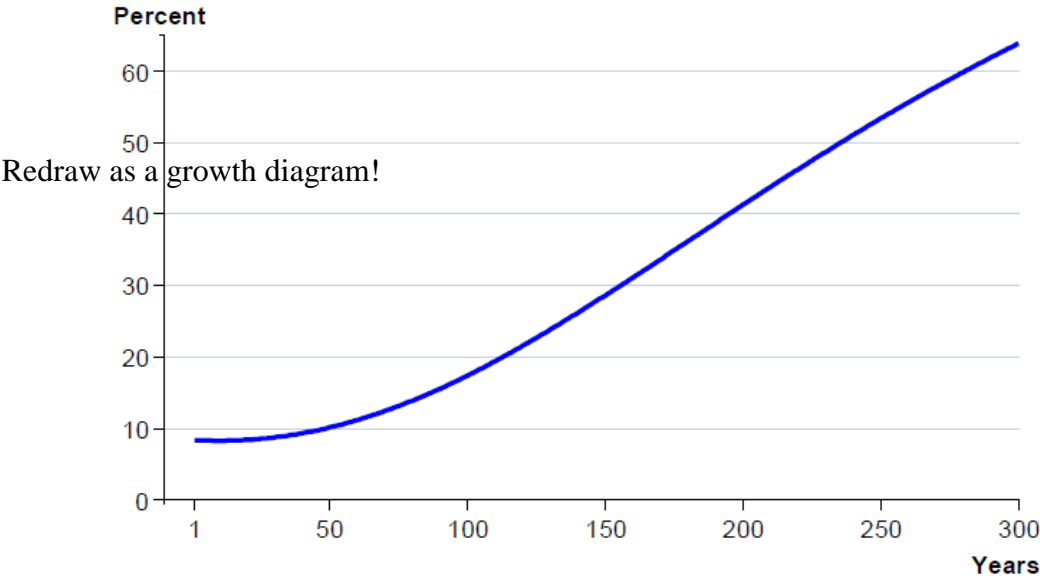
Notes: Growth rate of hypothetical economy over time. Other parameters:  $\zeta = 1.2$ ,  $\xi = 0.8$ ,  $\alpha = 0.6$ . Initial conditions:  $y(0) = 1,000$ ,  $x(0) = 0.6$ .

The blue line is the same as in the previous figure: both lines represent a parameterization with  $\psi = 0.4$  and  $\zeta = 1.2$ . Assuming a larger value of  $\psi$  implies that the capital gap gets a larger weight in the calculation of the growth rate, thereby generating an earlier and larger growth miracle. By contrast, assuming a lower value of  $\psi$  postpones and reduces the growth miracle. Holding constant the agglomeration externality and all other parameters, the intermediate value of  $\psi$  generates a growth path of the hypothetical economy that comes

close to the observed growth path.

The simulated growth path of the preferred parametrization implies a very low rate of convergence, far away from the "iron law" rate of (conditional) convergence in the vicinity of 2% (Barro 2015). With a convergence rate of 2%, halfway to the steady state would be reached after 35 years and 90% of an initial income gap would disappear after 115 years. These calculations reflect an early and large growth miracle predicted by the one-sector model. Figure 8 reveals that the hypothetical economy starting with a relative income just below 10% would need about 250 years to close half of the income gap with the leading economy, which is close to the halfway time of 280 years discussed in Section 4.2. To put this number into perspective: a halfway time of 250 years implies a rate of convergence of just 0.3%.

Figure 8. The simulated income path for the preferred parameterization



Notes: Relative GDP per person (%) of hypothetical economy over time. Parameters:  $\psi = 0.4$ ,  $\zeta = 1.2$ ,  $\xi = 0.8$ ,  $\alpha = 0.6$ . Initial conditions:  $y(0) = 1,000$ ,  $x(0) = 0.6$ .

## 6. Conclusions

The empirical analysis of a common transitional growth path is done by the technique of kernel regressions that does not impose a functional form on the data. Our kernels estimates show a hump-shaped transitional growth path, where the growth rate peaks at intermediate income levels. This result has a clear implication for theory of long-run growth. The one-sector steady state model, which predicts a log-linear transitional growth path, does not capture the essence of long-run development. We also show that a two-sector model can generate a hump-shaped transitional growth path.

Taken at face value, our empirical findings tell a story about the stylized long-run income dynamics of poor, rich, and middle-come countries, but the general pattern is overlaid by otherwise super-noisy data. Ignoring the noise, we find that the *poorest countries* have a low and unstable growth rate. With an average annual growth rate of about 1.6%, the static traditional steady state described by Malthus (1803) is all but gone, though there are a few countries with income levels that are lower in 2010 than in 1950.<sup>26</sup> The *richest countries* (excluding OPEC members) are also found to have an average annual growth rate of about 1.6%, but with a variation that is only about half as large as the one of the poorest countries.

*Middle-income countries* are found to have an average annual growth rate that is about one percentage point higher than the growth rate of the poorest and the richest countries. This holds over an income range from about 8 to 9 log points, which is also the income range when the variation of the growth rates starts to falls. Excess growth of one percentage point accumulates to half a log income point over a century. This means that many present middle income countries will slowly catch up with the high income countries during the next century, but reaching convergence will be a question of centuries rather than decades.

However, our stylized results are accompanied by an enormous variation of the growth rates at lower levels of income, which means that some countries may be able to escape a slow process of catching up by employing the institutions and policies that have been found to be positively correlated with long-run growth in numerous empirical studies. Of course the hard question remains why some countries have been capable of embarking on a more dynamic transitional growth path than others, especially at relatively low levels of income. But our contribution highlights that when searching for the deep roots of development in the data, it may make sense to allow at least for some cross-country commonality.

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26. This group includes Congo (Kinshasa), Central African Republic, Liberia, Madagascar, Niger, and Haiti.

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